PROJECT OUTLINE HANNIBAL'S LEMMA AND BEYOND

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Abstract. This document outlines a strategy to prove mode stability in Kerr de Sitter and beyond.

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1. The Idea

Let $\{a, M, Q, \Lambda\}$ be the parameters of a Kerr-Newman-de-Sitter spacetime. In the following we will be interested in the subextremal parameter range (whatever that is exactly in the de-Sitter case).

let $\{\omega_{rad}[a, M, Q, \Lambda, l, m]\}$ be the frequencies of radiating modes (that is solutions to the radial Teukolsky equation that are outgoing at the cosmological horizon and ingoing at the event horizon).

The set of all radiating modes is given by

$$\Omega[a, M, Q, \Lambda] := \{\omega_{rad}[a, M, Q, \Lambda, l, m] | l \in \mathbb{N}_0, m \in \mathbb{Z} \text{ with } |m| \leq l\} \tag{1.1}$$

we know that for Schwarzschild de Sitter we have

$$\{\omega \in \Omega[0, M, Q, \Lambda] | Im(\omega) \geq 0\} = \emptyset \tag{1.2}$$

To prove modestability for all of Kerr-Newman-de-Sitter one has to proceede in two steps:

1.1. Step 1.

Step 1. Establish continuous dependence of $\omega_{rad}[a, M, Q, \Lambda, l, m]$ on the parameters $\{a, M, Q, \Lambda > 0\}$

I have seen this statement claimed in the literature, but I never bothered looking up whether this has been proven on a satisfactory level of rigor.

Before we go to step two we note the following corollary:

Corollary 1. *Suppose there exists a spacetime with parameters* $\{a^*, M^*, Q^*, \Lambda^*\}$ *such that*

$$\{\omega \in \Omega[a^*, M^*, Q^*, \Lambda^*] | Im(\omega) \geq 0\} \neq \emptyset \tag{1.3}$$

then for every path

$$\gamma[\tau] \in \{a, M, Q, \Lambda\}, \tau \in [0, 1] \tag{1.4}$$

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in parameter space that connects to a Schwarzschild-de-Sitter spacetime $\{0, \tilde{M}, 0, \tilde{\Lambda}\}$*, there exists a unique parameter* τ_0 *such that for all* $\tau < \tau_0$ *we have that*

$$\{\omega \in \Omega[\gamma[\tau]] | Im(\omega) \geq 0\} = \emptyset \tag{1.5}$$

but

$$\{\omega \in \Omega[\gamma[\tau_0]] | Im(\omega) = 0\} \neq \emptyset. \tag{1.6}$$

Furthermore we have that for τ_0 *we have that*

$$\{\omega \in \Omega[\gamma[\tau_0]] | Im(\omega) > 0\} = \emptyset. \tag{1.7}$$

In particular this holds for the canonical path

$$\gamma[\tau] = \{\tau a*, M*, \tau Q*, \Lambda*\} \tag{1.8}$$

1.2. **Step 2.** Adapt the proof in Section 11.2 of [1] to show

Theorem 2. *For all parameters* $\{a, M, Q, \Lambda\}$ *in the subextremal range*

$$\{\omega \in \Omega[a_0, M_0, Q_0, \Lambda_0] | Im(\omega) > 0\} = \emptyset \tag{1.9}$$

implies

$$\{\omega \in \Omega[a_0, M_0, Q_0, \Lambda_0] | Im(\omega) \geq 0\} = \emptyset. \tag{1.10}$$

This implies that modestability holds for all subextremal Kerr-Newman-de-Sitter spacetimes that are path-connected to a SSdS spacetime in parameter space (where the entire path lies in the subextremal range) as together with the corollary it gives a contradiction.

Intuitively I would expect the domain of subextremal black hole spacetimes to be path-connected in parameter space (in particular I would expect the canonical path to be a choice that always works), but this has to be checked.

The case for $\omega = 0$ would likely have to be proven seperately as in the beginning of section 11 of their paper. (or generally the critical points)

References

[1] Felix Finster and Joel Smoller. Linear stability of the non-extreme kerr black hole. arXiv preprint arXiv:1606.08005, 2016.